

## Problem Set 4: Changes of State and Latent Heat

- 4.1 [a]  $m = 28.6 \text{ kg}, L_f = 3.34 \times 10^5 \text{ J kg}^{-1}$   
 $Q = mL_f = 28.6 \text{ kg} \times 3.34 \times 10^5 \text{ J kg}^{-1} = 9.55 \times 10^6 \text{ J}$   
 [b]  $m = 0.423 \text{ kg}, L_v = 2.26 \times 10^6 \text{ J kg}^{-1}$   
 $Q = mL_v = 0.423 \text{ kg} \times 2.26 \times 10^6 \text{ J kg}^{-1} = 9.56 \times 10^5 \text{ J}$   
 [c]  $m = 0.00458 \text{ kg}, L_v = 2.35 \times 10^6 \text{ J kg}^{-1}$   
 $Q = mL_v = 0.00458 \text{ kg} \times 2.35 \times 10^6 \text{ J kg}^{-1} = 1.08 \times 10^4 \text{ J}$
- 4.2  $Q = 1.85 \times 10^4 \text{ J}, m = 0.208 \text{ kg}$   
 $L_v = \frac{Q}{m} = \frac{1.85 \times 10^4 \text{ J}}{0.208 \text{ kg}} = 8.89 \times 10^4 \text{ J kg}^{-1}$
- 4.3  $Q = 9.53 \times 10^4 \text{ J}, L_f = 1.05 \times 10^5 \text{ J kg}^{-1}$   
 $m = \frac{Q}{L_f} = \frac{9.53 \times 10^4 \text{ J}}{1.05 \times 10^5 \text{ J kg}^{-1}} = 0.908 \text{ kg} = 908 \text{ g}$
- 4.4 [a] The latent heat absorbed when a liquid changes to a gas causes the cooling.  
 [b] The air conditioner actually removes heat from warm air. It then blows the cooled air into the car's cabin and the cooled air removes heat from the interior mainly by conduction. The cooling process involves compressing refrigerant gas, which heats up as it compresses. The hot compressed gas is fan-cooled and it condenses to liquid. The now cool liquid then has the pressure taken off and it partly evaporates. This cools the remaining liquid. Warm air is cooled by the cool refrigerant liquid.
- 4.5 If equal masses of water are involved in both incidents, more heat will be transferred to your hand by the steam because as well as transferring heat by cooling from  $100^\circ\text{C}$ , it also releases heat as it condenses on your hand (latent heat of vaporisation).
- 4.6 The heat energy from the iron is principally used to vaporise the water rather than to raise the temperature of Mario's finger.
- 4.7 [a] The mugs are both warmer than the ice and since heat energy always travels from 'hot to cold', they will transfer energy to their respective blocks of ice. As a result, the ice will absorb this released energy and begin a change of state phase (since it is already at  $0^\circ\text{C}$ ) and begin melting.  
 [b] As the glass cools from room temperature to  $0^\circ\text{C}$ , it releases heat energy at a greater rate (since it has a higher specific heat capacity) than the pewter and therefore melts the ice more quickly.  
 [c] Since pewter is an alloy of lead and tin, two metals and therefore good conductors of heat, this mug would conduct heat through its base at a greater rate than the glass mug meaning it would sink faster.
- 4.8 The lake has a large specific heat capacity due to the high specific heat capacity of water. This means that it cools down much more slowly than the surrounding earth. Heat energy flows from the relatively warmer lake to melt any nearby snow.

## Heating Processes

4.9 [a]  $m = 4 \text{ kg}, L_v = 2.42 \times 10^6 \text{ J kg}^{-1}$   
 $Q = mL_v = 4 \text{ kg} \times 2.42 \times 10^6 \text{ J kg}^{-1} = 9.68 \times 10^6 \text{ J} = 9.68 \text{ MJ}$   
 [b]  $Q = 9.68 \text{ MJ} = 9.68 \times 10^6 \text{ J}, m = 55 \text{ kg}, c = 3500 \text{ J kg}^{-1} \text{K}^{-1}$   

$$\Delta T = \frac{Q}{mc} = \frac{9.68 \times 10^6 \text{ J}}{55 \text{ kg} \times 3500 \text{ J kg}^{-1} \text{K}^{-1}} = 50.3^\circ \text{C}$$

[c] Air movement over your skin helps to evaporate perspiration from your skin. The latent heat needed for this phase change comes from your body. The combination of evaporation and forced convection cools you down.

[d] When out of the water, air movement or the movement of the swimmer results in significant amounts of water being evaporated from the swimmer's skin. The energy to do this is largely drawn from the swimmer's body, thus cooling the swimmer.

4.10 [a]  $s = ut + \frac{1}{2}at^2$   
 Since  $u = 0$ , then:  $s = \frac{1}{2} \times 9.8 \text{ ms}^{-2} \times (6 \text{ s})^2 = 176 \text{ m}$   
 [b]  $L_f \text{ for iron} = 2.76 \times 10^5 \text{ J kg}^{-1}, L_f \text{ for lead} = 2.5 \times 10^4 \text{ J kg}^{-1}$

The value for iron is 11 times greater than that of lead, which means that in order for iron to begin the solidification process it must lose 11 times the amount of energy per kilogram. This means that the tower would have to be a lot taller if iron ball bearings were to be made in this way.

This assumes that the surrounding air will take away heat from the bearings and that the iron will lose heat at a steady rate, that the air temperature will be constant and that the air pressure exerted on the ball bearing and the surface tension forces are sufficient to maintain the spherical shape, particularly at the higher altitude. It also assumes the iron ball bearing will keep accelerating until it hits the ground.

[c] Since air at a higher altitude is likely to be much cooler than air nearer the ground, the rate of loss of heat from the iron ball bearings would probably be greater initially so the tower could perhaps be made a little shorter. This is further reinforced by the fact that since iron has a higher melting point than lead, it does not have to cool down as much to begin solidifying. Finally, and maybe most significantly, since the tower would have to be much taller, the iron bearings would most certainly reach their terminal velocity well before they hit the pool of water. This means they would take longer to fall through the air, which is additional evidence that a smaller tower than that suggested above would do the job.

4.11 [a]  $m = 0.055 \text{ kg}, c_{\text{liquid}} = 105 \text{ J kg}^{-1} \text{K}^{-1}, \Delta T = 427^\circ \text{C} - 327^\circ \text{C} = 100^\circ \text{C}$   
 $Q = mc_{\text{liquid}}\Delta T = 0.055 \text{ kg} \times 105 \text{ J kg}^{-1} \text{K}^{-1} \times 100^\circ \text{C} = 578 \text{ J}$   
 [b]  $m = 0.055 \text{ kg}, L_f = 2.5 \times 10^4 \text{ J kg}^{-1}$   
 $Q = mL_f = 0.055 \text{ kg} \times 2.5 \times 10^4 \text{ J kg}^{-1} = 1.38 \times 10^3 \text{ J}$   
 [c]  $m = 0.055 \text{ kg}, c_{\text{solid}} = 130 \text{ J kg}^{-1} \text{K}^{-1}, \Delta T = 327^\circ \text{C} - 21.5^\circ \text{C} = 305.5^\circ \text{C}$   
 $Q = mc_{\text{solid}}\Delta T = 0.055 \text{ kg} \times 130 \text{ J kg}^{-1} \text{K}^{-1} \times 305.5^\circ \text{C} = 2.18 \times 10^3 \text{ J}$   
 [d]  $Q_{\text{total}} = 578 \text{ J} + 1.38 \times 10^3 \text{ J} + 2.18 \times 10^3 \text{ J} = 4.14 \times 10^3 \text{ J}$

4.12 [a]  $m = 0.0125 \text{ kg}, c = 4180 \text{ J kg}^{-1} \text{K}^{-1}, \Delta T = 100^\circ \text{C} - 26.5^\circ \text{C} = 73.5^\circ \text{C}$   
 Heat needed:  $Q = mc\Delta T = 0.0125 \text{ kg} \times 4180 \text{ J kg}^{-1} \text{K}^{-1} \times 73.5^\circ \text{C} = 3840 \text{ J}$   
 Time taken:  $t = \frac{Q}{P} = \frac{3840 \text{ J}}{48 \text{ J s}^{-1}} = 80 \text{ s}$   
 [b]  $m = 0.0125 \text{ kg}, L_v = 2.26 \times 10^6 \text{ J kg}^{-1}$   
 Heat needed:  $Q = mL_v = 0.0125 \text{ kg} \times 2.26 \times 10^6 \text{ J kg}^{-1} = 2.83 \times 10^4 \text{ J}$   
 Time taken:  $t = \frac{Q}{P} = \frac{2.83 \times 10^4 \text{ J}}{48 \text{ J s}^{-1}} = 590 \text{ s}$   
 [c] Total time =  $80 \text{ s} + 590 \text{ s} = 670 \text{ s}$

# Heating Processes

- 4.13  $m = 2.15 \text{ kg}, c = 4180 \text{ J kg}^{-1}\text{K}^{-1}, \Delta T = 21.5^\circ\text{C} - 0^\circ\text{C} = 21.5^\circ\text{C}$   
 Heat lost to cool water:  $Q = mc\Delta T = 2.15 \text{ kg} \times 4180 \text{ J kg}^{-1}\text{K}^{-1} \times 21.5^\circ\text{C} = 1.93 \times 10^5 \text{ J}$   
 $m = 2.15 \text{ kg}, L_f = 3.34 \times 10^5 \text{ J kg}^{-1}$   
 Heat lost to freeze the water:  $Q = mL_f = 2.15 \text{ kg} \times 3.34 \times 10^5 \text{ J kg}^{-1} = 7.18 \times 10^5 \text{ J}$   
 Total heat lost in 2 hours:  $Q_{total} = 1.93 \times 10^5 \text{ J} + 7.18 \times 10^5 \text{ J} = 9.11 \times 10^5 \text{ J}$   
 Rate of cooling:  $P = \frac{Q_{total}}{t} = \frac{9.11 \times 10^5 \text{ J}}{2 \text{ h} \times 3600 \text{ s h}^{-1}} = 127 \text{ J h}^{-1}$
- 4.14 The colder spaghetti will absorb some of the heat energy from the water thereby decreasing the temperature of the water.
- 4.15  $m = 0.455 \text{ kg min}^{-1}, c = 2010 \text{ J kg}^{-1}\text{K}^{-1}, \Delta T = 105^\circ\text{C} - 100^\circ\text{C} = 5^\circ\text{C}$   
 Heat supplied per second as steam cools:  
 $Q = mc\Delta T = (0.455 \text{ kg} \div 60 \text{ s}) \times 2010 \text{ J kg}^{-1}\text{K}^{-1} \times 5^\circ\text{C} = 76.2 \text{ J s}^{-1}$   
 $m = 0.455 \text{ kg min}^{-1}, L_v = 2.26 \times 10^6 \text{ J kg}^{-1}$   
 Heat supplied per second as steam condenses:  
 $Q = mL_v = (0.455 \text{ kg} \div 60 \text{ s}) \times 2.26 \times 10^6 \text{ J kg}^{-1} = 1.714 \times 10^4 \text{ J s}^{-1}$   
 Total heat supplied per second:  $P = 76.2 + 1.714 \times 10^4 = 1.72 \times 10^4 \text{ J s}^{-1}$
- 4.16 [a]  $Q = 55.6 \text{ MJ} = 5.56 \times 10^7 \text{ J}, m = 286 \text{ kg}, \Delta T = 452^\circ\text{C} - 22^\circ\text{C} = 430^\circ\text{C}$   
 $c_{alloy} = \frac{Q}{m\Delta T} = \frac{5.56 \times 10^7 \text{ J}}{286 \text{ kg} \times 430^\circ\text{C}} = 452 \text{ J kg}^{-1}\text{K}^{-1}$   
 [b]  
 $Q = 5.56 \times 10^7 \text{ J}, c_{water} = 4180 \text{ J kg}^{-1}\text{K}^{-1}, \Delta T = 100^\circ\text{C} - 22^\circ\text{C} = 78^\circ\text{C},$   
 $L_v = 2.26 \times 10^6 \text{ J kg}^{-1}$   
 Total heat taken away by water:  
 $Q = mc_{water}\Delta T + mL_v$   
 $5.56 \times 10^7 \text{ J} = m \times 4180 \text{ J kg}^{-1}\text{K}^{-1} \times 78^\circ\text{C} + m \times 2.26 \times 10^6 \text{ J kg}^{-1}$   
 Solving for m gives:  $m_{min} = 21.5 \text{ kg}$   
 [c] The steel does not lose heat any other way, eg. by radiation or by heating the surrounding air.
- 4.17 [a]  $m = 0.0232 \text{ kg}, c_{ice} = 2100 \text{ J kg}^{-1}\text{K}^{-1}, \Delta T_{ice} = 0^\circ\text{C} - (-10^\circ\text{C}) = 10^\circ\text{C}$   
 $L_f = 3.34 \times 10^5 \text{ J kg}^{-1}, c_{water} = 4180 \text{ J kg}^{-1}\text{K}^{-1}, \Delta T_{water} = 10^\circ\text{C} - 0^\circ\text{C} = 10^\circ\text{C}$   
 Heat absorbed to warm ice:  
 $Q = mc_{ice}\Delta T_{ice} = 0.0232 \text{ kg} \times 2100 \text{ J kg}^{-1}\text{K}^{-1} \times 10^\circ\text{C} = 487 \text{ J}$   
 Heat absorbed to melt ice:  
 $Q = mL_f = 0.0232 \text{ kg} \times 3.34 \times 10^5 \text{ J kg}^{-1} = 7749 \text{ J}$   
 Heat absorbed to warm water:  
 $Q = mc_{water}\Delta T_{water} = 0.0232 \text{ kg} \times 4180 \text{ J kg}^{-1}\text{K}^{-1} \times 10^\circ\text{C} = 970 \text{ J}$   
 Total amount of heat required:  $Q_{total} = 487 \text{ J} + 7749 \text{ J} + 970 \text{ J} = 9.21 \times 10^3 \text{ J} = 9.21 \text{ kJ}$   
 [b] The energy comes from the surrounding air and the glass itself.

# Heating Processes

- 4.18 [a]  $m_{BBQ} = 12 \text{ kg}$ ,  $c_{BBQ} = 445 \text{ J kg}^{-1}\text{K}^{-1}$ ,  $\Delta T_{BBQ} = 395^\circ\text{C} - 185^\circ\text{C} = 210^\circ\text{C}$   
 $c_{water} = 4180 \text{ J kg}^{-1}\text{K}^{-1}$ ,  $\Delta T_{water} = 100^\circ\text{C} - 20^\circ\text{C} = 80^\circ\text{C}$ ,  $L_v = 2.26 \times 10^6 \text{ J kg}^{-1}$   
 Heat removed from BBQ:  
 $Q = m_{BBQ}c_{BBQ}\Delta T_{BBQ} = 12 \text{ kg} \times 445 \text{ J kg}^{-1}\text{K}^{-1} \times 210^\circ\text{C} = 1.12 \times 10^6 \text{ J}$   
 Heat taken away by water:  
 $Q = m_{water}c_{water}\Delta T_{water} + m_{water}L_v$   
 $1.12 \times 10^6 \text{ J} = m_{water} \times 4180 \text{ J kg}^{-1}\text{K}^{-1} \times 80^\circ\text{C} + m_{water} \times 2.26 \times 10^6 \text{ J kg}^{-1}$   
 Solving for  $m_{water}$  gives:  $m_{water} = 0.432 \text{ kg} = 432 \text{ g}$   
 [b]  $L_f = 3.34 \times 10^5 \text{ J kg}^{-1}$ ,  $c_{water} = 4180 \text{ J kg}^{-1}\text{K}^{-1}$ ,  $\Delta T_{water} = 100^\circ\text{C}$ ,  
 $L_v = 2.26 \times 10^6 \text{ J kg}^{-1}$   
 Heat taken away by ice:  
 $Q = m_{ice}L_f + m_{ice}c_{water}\Delta T_{water} + m_{ice}L_v$   
 $1.12 \times 10^6 \text{ J} = m_{ice} \times 3.34 \times 10^5 \text{ J kg}^{-1} + m_{ice} \times 4180 \text{ J kg}^{-1}\text{K}^{-1} \times 100^\circ\text{C} + m_{ice} \times 2.26 \times 10^6 \text{ J kg}^{-1}$   
 Solving for  $m_{ice}$  gives:  $m_{ice} = 0.383 \text{ kg} = 383 \text{ g}$
- 4.19  $c_{ice} = 2100 \text{ J kg}^{-1}\text{K}^{-1}$ ,  $\Delta T_{ice} = 0^\circ\text{C} - (-11.3^\circ\text{C}) = 11.3^\circ\text{C}$ ,  $L_f = 3.34 \times 10^5 \text{ J kg}^{-1}$ ,  
 $\Delta T_{ice/water} = 3.6^\circ\text{C}$   
 $m_{water} = 0.195 \text{ kg}$ ,  $c_{water} = 4180 \text{ J kg}^{-1}\text{K}^{-1}$ ,  $\Delta T_{water} = 19.7^\circ\text{C} - 3.6^\circ\text{C} = 16.1^\circ\text{C}$   
 $m_{glass} = 0.215 \text{ kg}$ ,  $c_{glass} = 670 \text{ J kg}^{-1}\text{K}^{-1}$ ,  $\Delta T_{glass} = 19.7^\circ\text{C} - 3.6^\circ\text{C} = 16.1^\circ\text{C}$   
 Heat lost by water + heat lost by glass = heat gained by ice  
 Heat lost by water + heat lost by glass:  
 $Q = m_{water}c_{water}\Delta T_{water} + m_{glass}c_{glass}\Delta T_{glass}$   
 $Q = 0.195 \text{ kg} \times 4180 \text{ J kg}^{-1}\text{K}^{-1} \times 16.1^\circ\text{C} + 0.215 \text{ kg} \times 670 \text{ J kg}^{-1}\text{K}^{-1} \times 16.1^\circ\text{C}$   
 $Q = 1.54 \times 10^4 \text{ J}$   
 Heat gained by ice:  
 $Q = m_{ice}c_{ice}\Delta T_{ice} + m_{ice}L_f + m_{ice}c_{water}\Delta T_{ice/water}$   
 $1.54 \times 10^4 \text{ J} = m_{ice} \times 2100 \text{ J kg}^{-1}\text{K}^{-1} \times 11.3^\circ\text{C} + m_{ice} \times 3.34 \times 10^5 \text{ J kg}^{-1} + m_{ice} \times 4180 \text{ J kg}^{-1}\text{K}^{-1} \times 3.6^\circ\text{C}$   
 Solving for  $m_{ice}$  gives:  $m_{ice} = 0.0414 \text{ kg} = 41.4 \text{ g}$
- 4.20  $L_v = 2.26 \times 10^6 \text{ J kg}^{-1}$ ,  $c_{water} = 4180 \text{ J kg}^{-1}\text{K}^{-1}$ ,  $\Delta T_{steam/water} = 100^\circ\text{C} - 24.5^\circ\text{C} = 75.5^\circ\text{C}$   
 $m_{ice} = 1.5 \text{ kg}$ ,  $L_f = 3.34 \times 10^5 \text{ J kg}^{-1}$ ,  $\Delta T_{ice/water} = 24.5^\circ\text{C}$   
 Heat lost by steam/water = heat gained by ice/water  
 $m_{steam}L_v + m_{steam}c_{water}\Delta T_{steam/water} = m_{ice}L_f + m_{ice}c_{water}\Delta T_{ice/water}$   
 $m_{steam} \times 2.26 \times 10^6 \text{ J} + m_{steam} \times 4180 \text{ J kg}^{-1}\text{K}^{-1} \times 75.5^\circ\text{C} = 1.5 \text{ kg} \times 3.34 \times 10^5 \text{ J kg}^{-1} + 1.5 \text{ kg} \times 4180 \text{ J kg}^{-1}\text{K}^{-1} \times 24.5^\circ\text{C}$   
 Solving for  $m_{steam}$  gives:  $m_{steam} = 0.254 \text{ kg} = 254 \text{ g}$